NEW SOUTH WALES

Department of Education

LEAVING CERTIFICATE EXAMINATION, 1961

Mathematics II

PASS PAPER

Chief Examiner: T. G. ROOM, Sc.D.

Assessors: H. MULHALL, B.Sc., Ph.D.
E. S. ROLFE, B.Sc., Dip.Ed.

Time allowed—Three hours

Candidates may attempt all questions.

Question 1 carries 30 marks and the other questions 10 marks each.

Except in Question 1 marks will not be awarded to answers where the work is not shown.

Marks will be deducted for careless or badly arranged work.

Mathematical Tables and Squared Paper will be provided.

Slide rules may not be used.

Answers to the two Parts of this paper are to be returned in *separate books* marked A or B.

Part A

1. (i) Write down the equation of the line joining the origin to the point of intersection of the lines

$$2x + 3y + 1 = 0$$
$$x - y + 5 = 0.$$

(ii) The line $2x + y = 4$ meets the x and y axes respectively in A and B. C is the point on the x-axis such that ABC is an isosceles triangle with AC as base. Find the equation of BC.

(iii) If $\tan\theta = \dfrac{a^2 - b^2}{2ab}$, what are the possible values of $\cos\theta$?

(iv) Without using the tables, find as a surd in its simplest form, the value of

$$\frac{\sin 70^\circ + \sin 20^\circ}{\cos 25^\circ}.$$

(v) Find the equation of the directrix of the parabola

$$12y = x^2 - 8x + 4.$$

(vi) Find the equation of the normal to the parabola

$$12y = x^2 - 8x + 4$$

at the point $(1, -\frac{1}{4})$.

(vii) Given that

$$\sin\theta + \sqrt{3}\cos\theta = \sqrt{2},$$

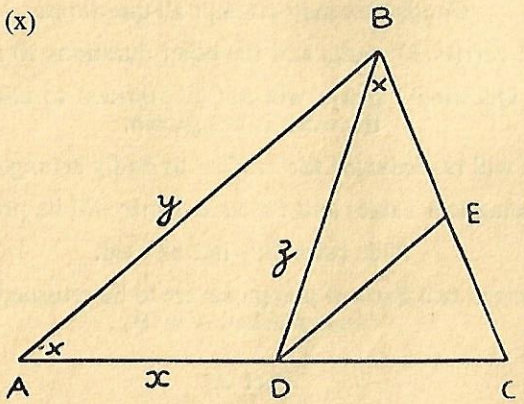
that θ is measured in radians and $0 < \theta < \frac{1}{2}\pi$, find θ without using tables.

(viii) Find the coordinates of the point of the line

$$x = 3t - 5$$
$$y = 2t + 1$$

that is closest to the origin.

(ix) and (x)



The points D, E on the sides AC, BC of the triangle ABC are such that

$$\angle BAD = \angle CBD,$$
DE is parallel to AB,
$$AD = x,\ AB = y,\ DB = z.$$

Find, in terms of x, y, z:—

(ix) The length of—

(a) DE,

(b) DC.

(x) The ratio of the areas—

(a) $\dfrac{\triangle DEC}{\triangle ABC}$,

(b) $\dfrac{\triangle ABD}{\triangle ABC}$.

2. P_1 is the point (x_1, y_1), and P_2 is the point (x_2, y_2). Show that the coordinates of any point P on the line P_1P_2 may be expressed in the form

$$\left(\frac{x_1 + kx_2}{1 + k},\ \frac{y_1 + ky_2}{1 + k}\right).$$

State for what range of values of k, P lies between P_1 and P_2.

A line through the point F, (3, 2), meets the x-axis in C, $(c, 0)$, and the y-axis in D, $(0, d)$, and $CF : FD = k : 1$. Express c and d in terms of k, and draw a sketch to show the positions of C and D when—

(a) $k = 2$,

(b) $k = -2$.

3. (i) Find, as accurately as your tables permit, the angles between 0° and 360° which satisfy the equation

$$10\sin^2\theta + \cos\theta - 7 = 0.$$

(ii) If $\tan\alpha$ and $\tan\beta$ are the two values of $\tan\theta$ which satisfy the equation

$$a\tan^2\theta + b\tan\theta + c = 0,$$

find $\tan(\alpha + \beta)$, and show that

$$\tan^2(\alpha - \beta) = \frac{b^2 - 4ac}{(a + c)^2}.$$

Part B

4. A and B are the points $(-a, 0)$ and $(a, 0)$ respectively. P is a variable point which moves so that PA = 2PB. Show that the locus of P is a circle, and write down the coordinates of its centre and the length of its radius.

If O is the origin, C the centre of the circle and Q the point in which the circle meets AB internally, show that BC = 2OQ.

5. A man walking along a straight road in a horizontal plane observes a tower on his left from two points A and B which are 400 feet apart. From A the tower is due north and the elevation of the top of it is 26° 32′. From B it is due west and the elevation of the top is 22° 14′. Find—

(a) the height of the tower, to the nearest foot;

(b) the bearing of B from A, to the nearest minute.

6. S is the point $(a, 0)$, M is the foot of the perpendicular from a variable point P to the line $x + a = 0$, and P moves so that SP = MP. Find the equation of the locus of P.

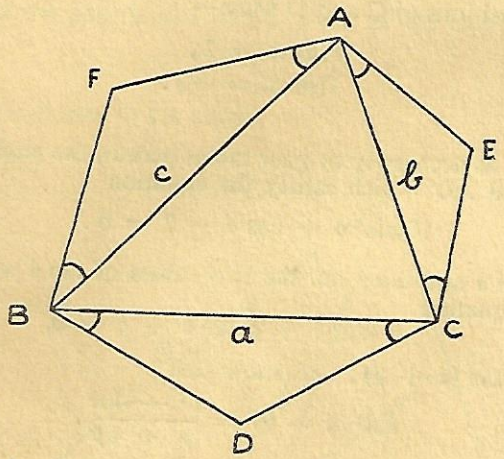
Q is the point on MP produced such that MP = PQ. Find the equation of the locus of Q.

7. ABC is an acute angled triangle, having sides a, b, c. Prove that—

(i) $a^2 = b^2 + c^2 - 2bc \cos A$;

(ii) the area, Δ, of the triangle is given by

$$\Delta = \tfrac{1}{2}bc \sin A.$$



In the diagram above ABC is an acute-angled triangle, and D, E, F are the points such that each of the angles ∠DBC, ∠DCB, ∠ECA, ∠EAC, ∠FAB, ∠FBA is 30°.

By using the cosine formula, or otherwise, show that

$$EF^2 = \frac{1}{6}(a^2 + b^2 + c^2) + \frac{2\sqrt{3}}{3}\Delta.$$

8. A line L passes through the point P, $(h, 0)$, making an angle α with the x-axis. Write down expressions for the coordinates of a variable point on the line whose (signed) distance from P is r.

If the line L meets the parabola $y^2 = 4ax$ in the points Q and R, show that the signed distances, PR, PQ, are the roots of the equation

$$r^2 \sin^2 \alpha - 4ar \cos \alpha - 4ah = 0,$$

and that the coordinates of M, the mid-point of QR, are

$$(h + 2a \cot^2 \alpha,\ 2a \cot \alpha).$$

If P remains fixed, find the equation of the locus of M as α varies.

NEW SOUTH WALES

Department of Education

LEAVING CERTIFICATE EXAMINATION, 1961

Physics

PASS PAPER

Chief Examiner: Dr. MALCOLM FRASER, M.A.
Assessors : Dr. W. H. LOVE, B.Sc.
Mr. D. M. HENDERSON, M.Sc.

Time allowed—Three hours

Two questions to be answered from each part.

Question 7 is compulsory.

All questions are of equal value.

The number of significant figures given in every numerical answer must be consistent with those given in the numerical data.

Mathematical tables and graph paper will be provided. Drawing instruments and slide rules may be used.

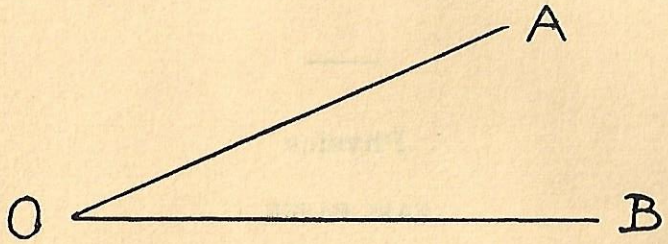
Answers are to be returned in four separate books, each book containing answers to questions from one part only and each being appropriately labelled.

43365—111

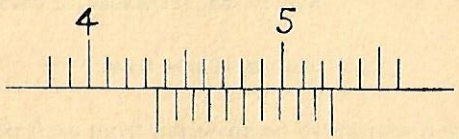
Part A

Two questions to be answered from this part.

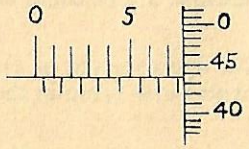
1. (*a*) (i) Measure the angle AOB expressing your answer in radians.



(ii) What is the reading on the vernier? (The name of the unit is not required.)



(iii) What is the reading on the micrometer screw gauge? (The name of the unit is not required.)



(*b*) From a height of 16 ft. above the ground a stone is projected upwards with a velocity of 24 ft. per sec. Determine the time it takes to reach the ground, taking $g = 32$ ft. per sec. per sec. What is the velocity of the stone at the instant before it reaches the ground?

(*c*) Distinguish between 1 gm. and 1 gm. wt.

2. (*a*) Name the standards of length, mass and time in the c.g.s. system of units. Briefly state how the standard of time is obtained.

(*b*) A sphere of mass 500 gm. hangs from one end of a uniform steel wire of unstretched length 200·03 cm., the upper end of which is attached to a fixed point. The length of the stretched wire is measured and found to be 200·08 cm.

A horizontal force F is applied to the sphere and in the equilibrium position the wire makes an angle of 30° with the vertical. Draw a diagram showing all the forces acting on the sphere in this equilibrium position. Find the tension in the wire, its new length and the value of F.

3. (*a*) A car is travelling along a rough horizontal road at a constant velocity. Draw a side view of the car and mark with the letter D the wheel which is driving the car. On your diagram show—

(i) the direction in which the car is moving;

(ii) the directions of the frictional forces between the tyres and the road.

What is the resultant of all the external forces acting on the car? Justify your answer.

(*b*) The external diameter of the tyres of a car is 25 in.; what is the angular velocity of the wheels in radians per second when the car is travelling at 25 miles per hour?

(*c*) Car A of mass 2,000 pounds, travelling at a constant velocity of 40 miles per hour along a horizontal road, comes to a long smooth section, where it collides with car B of mass 1,000 pounds, which is proceeding in the same direction with a velocity of 10 miles per hour. Given that the velocity of B is 30 miles per hour after the collision but that its direction is unchanged, determine the velocity of A after the collision.

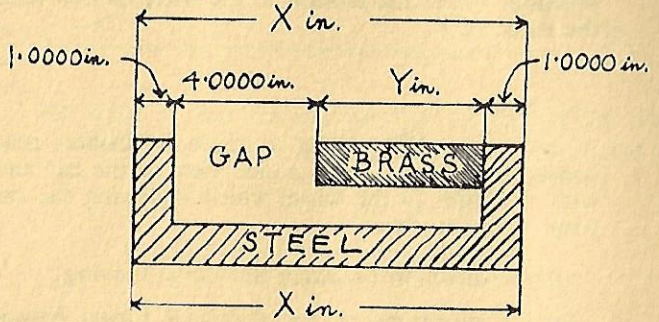
Part B

Two questions to be answered from this part.

4. (*a*) Define the British Thermal Unit.

Given that 1·00 Kg. = 2·20 lb., determine the value of 1 B.T.U. in calories.

(*b*) The diagram, which is not drawn to scale, shows a gauge made of steel and brass. The width of the gap is 4·0000 in. What must be the values of X and Y in order that the width of the gap may not change when the temperature of the room changes? The coefficients of linear expansion for brass and steel are $18{\cdot}0 \times 10^{-6}$ and $10{\cdot}8 \times 10^{-6}$ per centigrade degree respectively.



5. (*a*) Draw a neat well labelled diagram of an apparatus used by Joule to determine the mechanical equivalent of heat.

(*b*) For a certain wet and dry bulb hygrometer it has been found that the partial pressure p mm. of the water vapour present in the air is given by the formula

$$p = P - 0{\cdot}60\ (\theta_D - \theta_W)$$

where θ_D° C. = temperature recorded by the dry bulb;
θ_W° C. = temperature recorded by the wet bulb;
P mm. = sat. vapour pressure at temp. θ_W° C.

Given that the saturated vapour pressure of water varies with temperature as shown in the following table:—

Temp. in °C.	7·0	10·0	13·0	16·0	19·0
S.V.P. in mm.	7·51	9·21	11·23	13·62	16·46

determine the relative humidity and the dew point when the dry bulb reads 17·9° C. and the wet bulb reads 12·4° C.

6. Write brief notes on each of the following topics:—

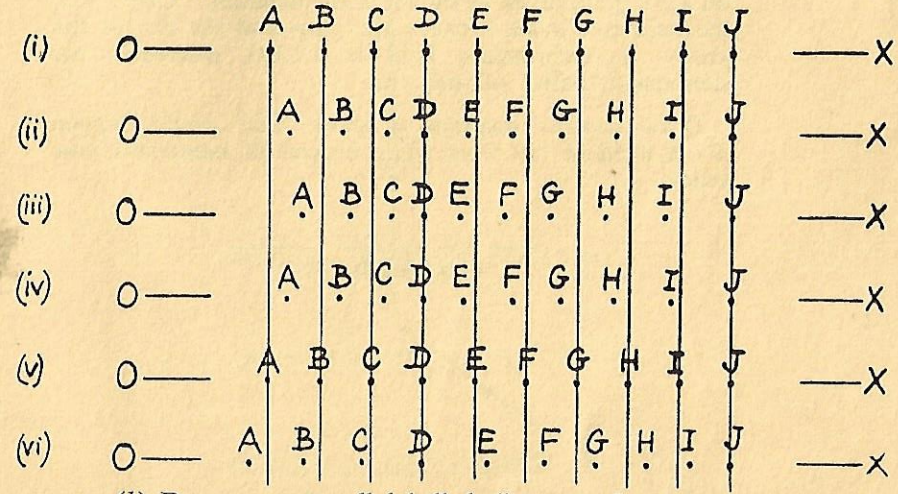
(*a*) Thermionic emission.
(*b*) X-rays.
(*c*) Fusion as a source of energy.
(*d*) Fission as a source of energy.

(Candidates are advised to spend not more than about five minutes on each topic.)

Part C

Two questions must be answered from this part. One of the two questions must be question 7.

7. (*a*) In diagram (i) A, B, C . . . J are the positions occupied by small quantities of air which lie on the line OX. When this air is vibrating, diagram (ii) shows the positions of these quantities of air at a given instant and diagrams (iii), (iv), (v) and (vi) show the positions of these quantities of air at successive intervals of $3{\cdot}0 \times 10^{-5}$ seconds. By measuring with a centimetre scale determine the maximum amplitude of the vibration and the wave length. Show that the period of the vibration is $24{\cdot}0 \times 10^{-5}$ seconds. What is the frequency of the vibration and what is the velocity of sound in the air? Name the kind of waves illustrated and suggest a method for causing the air to vibrate in the manner shown.



(*b*) Draw a neat well labelled diagram of an apparatus you could use for determining the velocity of sound in air.

8. (*a*) A thin glass lens has a power of +5·00 dioptres. State whether it is convex or concave and determine its focal length.

A linear object AB of length 2·0 cm. is perpendicular to the axis of the lens with B on the axis and distant 40·0 cm. from the centre of the lens. Determine the length of the image.

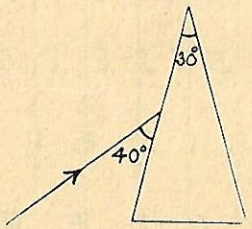
The object is rotated through a right angle about B, so that AB is on the axis with A distant 38·0 cm. from the centre of the lens. What is the length of its new image?

(*b*) A man standing in a paddock sees two trees P and Q, P being slightly to the left of Q. He walks several paces to his right and in his new position observes that P is slightly to the right of Q. What information can you give about the relative distances of P and Q from the man?

What is the name given to this method of comparing distances?

9. (*a*) A monochromatic ray of yellow light in air is incident on a 30° glass prism as shown in the diagram. Given that the refractive index between the glass and the air for the yellow monochromatic light is 1·5320, determine the subsequent paths of the ray.

Draw another diagram to show what would happen if the incident ray were white instead of monochromatic yellow.

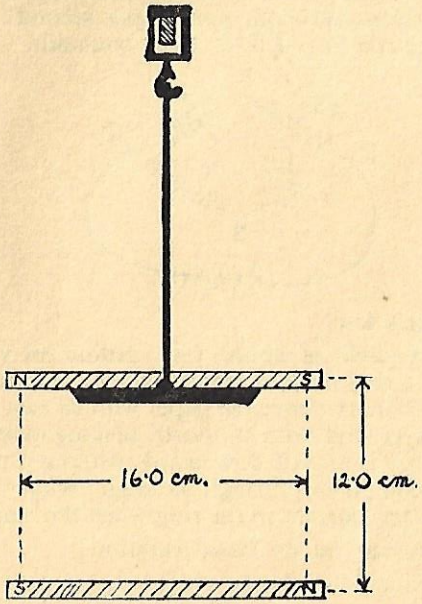


(*b*) Describe briefly what you understand by selective absorption.

Part D

Two questions to be answered from this part.

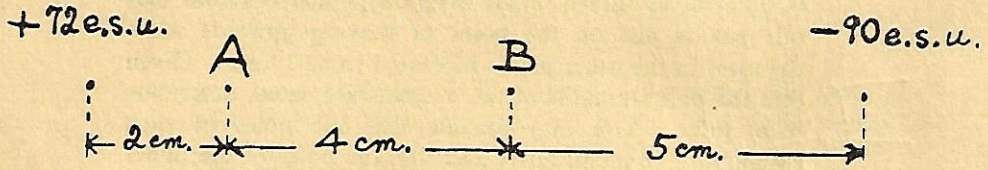
10. A bar magnet is placed on the pan of a beam balance and when masses amounting to 60·75 gm. are placed in the other pan equilibrium is attained. An identical magnet is placed under the pan on which the first magnet is situated, as shown in the diagram, and it is found that this pan is just on the point of moving upwards when the mass in the other pan is increased to 62·19 gm. Given that the pole strengths of the magnets are equal, determine their value. You may assume that the poles of each magnet are 16·0 cm. apart and that the poles of the upper magnet are 12·0 cm. vertically above those of the lower one.



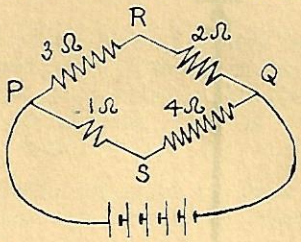
11. (*a*) Draw a neat well labelled diagram of each of the following:—
(i) an electromagnet; (ii) an electroscope.

(*b*) Point charges of +72 e.s.u. and —90 e.s.u. are situated as shown in the diagram. What is the intensity of the electric field at A. If an electron were placed at A what force would act on it?

Given that the potentials at A and B are +26 and —6 e.s.u. respectively, calculate the work required to move an electron from A to B. The charge on an electron is $-4{\cdot}8 \times 10^{-10}$ e.s.u.



12. (*a*) The potential difference between P and Q is 10 volts, P being at the higher potential. Determine the current in each resistance and the potential difference between R and S. Which is at the higher potential, R or S? How many electrons leave the battery per second? The charge on an electron is $-1{\cdot}6 \times 10^{-19}$ coulomb.



(*b*) State Lenz's law.

Draw a circle of about 1 in. radius on your paper to represent a ring made of copper wire. A small bar magnet is held stationary above the paper with its axis perpendicular to the paper and with its north seeking pole downwards. The magnet is moved downwards without rotation so that it passes completely through the ring. What is the direction of the electric current in the ring when the magnet is—

(i) stationary in its initial position;
(ii) moving towards the ring;
(iii) moving downwards from the ring after having passed through it?

NEW SOUTH WALES

Department of Education

LEAVING CERTIFICATE EXAMINATION, 1961

Physics

HONOURS PAPER

Chief Examiner: Dr. MALCOLM FRASER, M.A.

Assessors: Dr. W. H. LOVE, B.Sc.
Mr. D. M. HENDERSON, M.Sc.

Time allowed—Three hours

One question to be answered from each of Parts A, B and C, and *two* questions to be answered from Part D. *One* of the *two* questions from Part D must be question 9.

A separate book is to be used for the answers to each part. Each book must be appropriately labelled.

All questions are of equal value.

The number of significant figures given in every numerical answer must be consistent with those given in the numerical data.

Mathematical tables and graph paper will be provided.

Slide rules and drawing instruments may be used.

43365—119

Part A

One question to be answered from this part.

1. (*a*) A horizontal force F acts on a body of mass 100 gm. and gives it an acceleration of 8·0 cm. per sec. per sec. up a rough plane inclined at an angle of 45° to the horizontal. Given that the coefficient of friction between the body and the plane is 0·20, determine the magnitude of F.

(*b*) A small bubble of gas 0·20 mm. diameter is 9·80 cm. below the surface of a liquid of density 1·02 gm. per cc. If the atmospheric pressure is 760 mm. determine the pressure inside the bubble.

The surface tension of the liquid is 50 dynes per cm.

2. (*a*) Assuming that the earth is a sphere of radius 4,000 miles, calculate the acceleration due to gravity at a height of 200 miles above the earth's surface. The value of g at the earth's surface is 32·2 ft. per sec. per sec.

A sputnik is moving in a circular orbit at a constant height of 200 miles above the earth's surface. Determine the period of the sputnik.

(*b*) State the dimensions of—

sin θ, pressure, kinetic energy, strain, surface tension.

Part B

One question to be answered from this part.

3. (*a*) A copper sphere, whose surface is blackened, has a radius of 2·00 cm. and a mass of 300 gm. The sphere is suspended by a thin thread in bright sunlight on a still day when the air temperature is 20·0° C. The temperature of the sphere is found to vary with time as shown by the following table:—

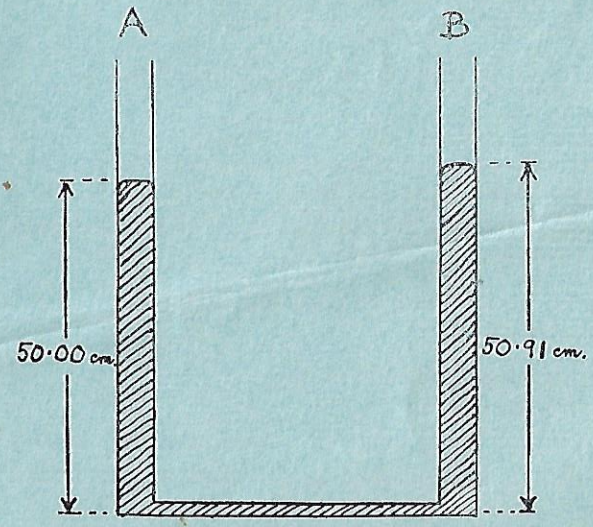
Time in minutes ..	0	1·0	2·0	3·0
Temperature in °C. ..	20·00	20·93	21·83	22·70

Assuming that the sphere absorbs all the radiation which falls upon it and assuming that none of the sun's radiation is absorbed by the atmosphere, determine the rate in watts, at which the sun is radiating energy. At what rate is matter being annihilated in the sun?

Specific heat of copper = 0·09.

Distance of the sun from the earth = $1{\cdot}5 \times 10^{13}$ cm.

(*b*) Two vertical glass tubes A and B, each of equal cross sectional area and connected by a horizontal glass tube of narrow bore, contain mercury as shown in the sketch. A and B are maintained at 0·0° C. and 100·0° C. respectively by ice-water and steam jackets. Determine the coefficient of cubical expansion of mercury. The coefficient of linear expansion of glass is 0·000008 per C. degree.



4. (*a*) The curved surface of a cylindrical bar of metal is thermally insulated from the surroundings and the ends of the bar are maintained at different but constant temperatures. What are the factors which determine the rate, R, at which heat is conducted along the bar and in what way does R depend upon each of these factors?

(*b*) Determine the root mean square velocity of oxygen molecules at 20° C.

Molecular weight of oxygen = 32.

Gram molecular volume at S.T.P. = 22·4 litres.

(*c*) By making any suitable assumptions, so as to simplify the calculations, determine the number of molecules of oxygen which hit one square centimetre of the wall of the containing vessel in one second when the vessel is filled with oxygen at 20° C. and 760 mm. Any assumptions made must be stated.

Mass of a molecule of oxygen = $5{\cdot}3 \times 10^{-23}$ gm.

Part C

One question to be answered from this part.

5. (*a*) Draw a diagram of a spectroscope and show the paths followed by some rays of light through the system when an incandescent lamp is being observed.

State the function of each part of the spectroscope you have drawn.

(*b*) Draw what would be observed in a spectroscope which is illuminated by—

(i) an incandescent lamp;

(ii) the sun;

(iii) a mercury lamp, or a neon lamp, or a sodium flame;

(iv) an incandescent lamp which is surrounded by red glass.

(*c*) The faintest sound which can be heard by the human ear has an energy flow of $1{\cdot}0 \times 10^{-16}$ watts per sq. cm. What must be the amplitude of sound waves, with a frequency of 316 per second, which can just be heard? You

may assume that the energy of a mass m undergoing simple harmonic motion of amplitude a and frequency n is $2\pi^2 ma^2 n^2$. The density of air is 1·20 gm. per litre and the velocity of sound in air is $3{\cdot}4 \times 10^4$ cm. per sec.

(*d*) Draw a rough graph to show how the amplitude of vibration of a tuning fork varies with time.

6. (*a*) By means of diagrams show what you understand by the following defects of a lens:—

(i) longitudinal spherical aberration;

(ii) longitudinal chromatic aberration.

With the aid of labelled diagrams and few words explain a method of overcoming each defect.

(*b*) A thin convex lens of low power is placed on a sheet of plate glass. The arrangement is illuminated first from above and then from below, and the region where the lens touches the glass is viewed from above.

Draw diagrams to illustrate what would be observed when the source of illumination is:

(i) a sodium flame;

(ii) an incandescent lamp.

Separate diagrams must be drawn for each of the four cases.

(*c*) Answer *either* (i) *or* (ii).

(i) Explain fully why the hole in a pinhole camera should be neither too large nor too small.

Or,

(ii) A small coin is held out at arm's length at night by an observer who, with one eye shut, interposes the coin between his open eye and a distant light. He sees the coin as a dark disc with a bright circular edge. Explain the reason for this.

Part D

Two questions to be answered from this part. One of the *two* questions must be question 9.

7. (*a*) Draw a neat well labelled diagram showing the essential parts of a moving coil galvanometer.

Derive a relation between the deflection of the coil and the current flowing in it.

(*b*) A galvanometer has a resistance of 0·01 ohm and the full scale deflection is obtained when a current of 50 milliamperes flows in the coil. Show how the galvanometer can be converted into—

(i) an ammeter with a range of 0—5 amps;
(ii) a voltmeter with a range of 0—10 volts.

(*c*) Draw a neat well labelled diagram of a diode valve. By means of a circuit diagram show how such a diode can be used as a rectifier.

8. (*a*) The plate and the case of an electroscope are connected to the negative and the positive terminals respectively of a 6-volt battery. State what is observed and give a reason for your answer.

(*b*) State the factors which determine the capacity of a condenser. In what way does the capacity depend upon each of these factors?

(*c*) A parallel plate condenser of capacity C farads consists of two plates A and B which are close together. B is connected to earth and a charge of +Q coulombs is given to A. What is—

(i) the charge on B;
(ii) the potential of A;
(iii) the potential of B.

(*d*) Three uncharged parallel plate condensers of capacities C_1, C_2 and C_3 farads respectively are joined in series in the order given. The outer plate of the third condenser is earthed and a charge is given to the outer plate of the first condenser so that the potential of this plate is V_1 volts. Draw a diagram of the arrangement and determine the potential of, and the charge on, each of the six plates.

(*e*) State Faraday's laws of electrolysis.

9. (*a*) Briefly describe experiments on—

(i) the scattering of α particles;
(ii) the disintegration of nitrogen by α particles.

(*b*) Give a brief account of the electromagnetic wave spectrum. State the approximate wave lengths of the waves in the several regions and briefly describe how waves of these wave lengths may be produced.

(*c*) Answer any four of the parts (i) to (v). Who discovered—

(i) the photoelectric effect,
(ii) neutrons,
(iii) isotopes,
(iv) radioactivity,
(v) X-rays?

(*d*) Given that the charge on an electron is $-1{\cdot}6 \times 10^{-19}$ coulombs and that 1 faraday = 96,500 coulombs, determine the number of molecules in 32 gm. of oxygen. Hence determine the mass of the hydrogen atom.

Molecular weight of oxygen = 32.

Atomic weight of hydrogen = 1·008.